

Precalculus

I. Functions

A. Characteristics and Representations of Functions

1. Describe parent functions symbolically and graphically, including $y = x^n$, $y = \ln x$, $y = \log_a x$, $y = 1/x$, $y = e^x$, $y = a^x$, $y = \sin x$ and the other trigonometric functions.
2. Using graphs, tables, and symbols, determine the domain and range for each of the functions studied.
3. Know the definition of sine and cosine functions based on a right triangle and on a unit circle as a wrapping function.
4. Evaluate all six trigonometric functions using the right triangle and wrapping function definitions.
5. Describe symmetry of graphs of even and odd functions.
6. Recognize and use connections among significant points of a function (roots, maximum points, and minimum points), the graph of a function, and the algebraic representation of a function.
7. Investigate continuity, end behavior, vertical and horizontal asymptotes, and limits; connect these characteristics to the graph of a function.

B. Operations on Functions.

1. Apply basic transformations, including $a \cdot f(x)$, $f(x) + d$, $f(x - c)$, $f(b \cdot x)$, $|f(x)|$, $f(|x|)$, to the parent functions.
2. Perform operations including composition and decomposition on functions, find inverses, and describe these procedures and results verbally, numerically, algebraically, and graphically.
3. Investigate identities graphically and verify them algebraically, including logarithmic properties, trigonometric identities, and exponential properties.

II. Functions as Models

A. Polynomial and Rational Functions

1. Use quotients to describe the graphs of rational functions, describe limitations on the domains and ranges, and examine asymptotic behavior.
2. Analyze various representations of polynomial and rational functions with respect to problem situations.
3. Solve polynomial and rational equations and inequalities using graphs, tables, and algebraic methods by using paper-and-pencil computations, graphing calculators, computer algebra systems, and spreadsheets.
4. Analyze a situation modeled by a polynomial and rational function, formulate an equation or inequality, and solve the problem.

Precalculus

B. Exponential and Logarithmic Functions

1. Develop the definition of logarithms by exploring and describing the relationship between exponential functions and their inverses.
2. Use the parent functions to investigate, describe, and predict the effects of parameter changes on the graphs of exponential and logarithmic functions, describe limitations on the domains and ranges, and examine asymptotic behavior.
3. For given contexts, determine the reasonable domain and range values of exponential and logarithmic functions, as well as interpreting and determining the reasonableness of solutions to exponential and logarithmic equations and inequalities.
4. Solve exponential and logarithmic equations and inequalities using graphs, tables, and algebraic methods by using paper-and-pencil computations, graphing calculators, computer algebra systems, and spreadsheets.
5. Analyze a situation modeled by an exponential or logarithmic function, formulate an equation or inequality, and solve the problem.
6. Interpret rates of change as they apply to phenomena such as inflation, spread of disease, population growth, tax brackets, pollution, and so forth.
7. Analyze graphical data gathered by technical equipment including combinations of graphs, periodic phenomena, and relative rates of change.

C. Trigonometric Functions

1. Use trigonometric functions to model real-life data.
2. Apply sine and cosine functions to periodic phenomena.
3. Solve applied problems using the law of sines and law of cosines.

III. Advanced Mathematics Topics

A. Sequences and Series

1. Represent patterns using arithmetic and geometric sequences and series, including the use of sigma notation to represent series.
2. Use arithmetic, geometric, and other sequences and series to solve real-life problems.
3. Apply informal concepts of successive approximation, upper and lower bounds, and limit in measurement situations: estimate lengths of curves, areas of curved regions, and volume of curved solids.
4. Describe limits of sequences and apply their properties to investigate convergent and divergent series.
5. Apply sequences and series to solve problems including sums, binomial expansion, the binomial theorem, combinations, and Pascal's triangle.
6. Prove statements about sequences and series using the principle of mathematical induction.

Precalculus

B. Conic Sections, Parametric Representations, and Polar Representations

1. Use conic sections to model motion, such as the graph of velocity vs. the position of a pendulum and motions of planets.
2. Use properties of conic sections to describe physical phenomena such as the reflective properties of light and sound.
3. Convert between parametric and rectangular forms of functions and equations to graph them.
4. Use parametric functions to simulate problems involving motion.
5. Convert between polar and rectangular forms of functions and graph them.

C. Vectors

1. Use the concept of vectors to model situations defined by magnitude and direction.
2. Use vectors to represent situations that involve both magnitude and direction, such as force, displacement, velocity, and acceleration.
3. Draw a pair of perpendicular vectors to find a distance graphically.